4.9) $A_1 = \begin{bmatrix} -5 & 1 & -1 \\ 15 & 0 & 3 \\ 7 & -1 & 3 \end{bmatrix}$

$P(\lambda) = \det(\lambda I - A_1) = \det\left(\begin{bmatrix} \lambda+5 & -1 & 1 \\ -15 & \lambda & -3 \\ -7 & 1 & \lambda-3 \end{bmatrix}\right) =$

$= (\lambda+5).\left[(\lambda^2-3\lambda)+3\right] + \left[(-15\lambda+45)-21\right] + (-15+7\lambda) =$

$= \lambda^3 - 3\lambda^2 + 3\lambda + 5\lambda^2 - 15\lambda + 15 - 15\lambda + 24 - 15 + 7\lambda =$

$= \lambda^3 + 2\lambda^2 - 20\lambda + 24$

Autoval. $\rightarrow$ $P(\lambda) = 0$ $\rightarrow$ $\left.\begin{matrix}\lambda_1 = 2 \\ \lambda_2 = 2\end{matrix}\right\}$ M.alg. $= 2$

$\rightarrow \lambda_3 = -6 \}$ M.alg. $= 1$

Para $\lambda = 2$

$\begin{pmatrix} 7 & -1 & 1 \\ -15 & 2 & -3 \\ -7 & 1 & -1 \end{pmatrix} \begin{matrix} \\ F2 \rightarrow 15F1 + 7F2 \\ F3 \rightarrow F1 + F3 \end{matrix} \begin{pmatrix} 7 & -1 & 1 \\ 0 & -1 & -6 \\ 0 & 0 & 0 \end{pmatrix} \rightarrow \begin{matrix} 7x - y + z = 0 \; (*) \\ -y - 6z = 0 \rightarrow y = -6z \end{matrix}$

$(*) \rightarrow 7x + 7z = 0 \rightarrow x = -z$

$\rightarrow \bar{x} = (-z; -6z; z) = \underbrace{z.(-1; -6; 1)}_{\text{MULT. GEOM.} = 1 < \text{MULT. ALG} = 2}$

~~[illegible]~~

Para $\lambda = -6$

$$\begin{pmatrix} -1 & -1 & 1 \\ -15 & -6 & -3 \\ -7 & 1 & -9 \end{pmatrix} \begin{matrix} \\ F2 \to 15F1 - F2 \\ F3 \to 7F1 - F3 \end{matrix} \begin{pmatrix} -1 & -1 & 1 \\ 0 & -9 & 18 \\ 0 & -8 & 16 \end{pmatrix} F3 \to 8F2 - 9F3$$

$$\begin{pmatrix} -1 & -1 & 1 \\ 0 & -9 & 18 \\ 0 & 0 & 0 \end{pmatrix} \to \begin{cases} -x-y+z=0 \to -x-z=0 \to x=-z \\ -9y+18z=0 \to y=2z \end{cases}$$

$$\to \bar{x} = (-z; 2z; z) = \underbrace{z.(-1;2;1)}_{\text{M.geom.} = 1 \checkmark}$$

$$J = \begin{bmatrix} 2 & 1 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & -6 \end{bmatrix}.$$

$$A_1 . P = P . J$$

$\to$ siendo $P = [v1, v2, v3]$ $\begin{cases} v1 = (-1, -6, 1) \\ v3 = (-1, 2, 1) \end{cases}$

$$\begin{cases} A_1 v1 = 2v1 \\ A_1 v2 = [v1, v2\ v3] . \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} = v1 + 2v2 \to \boxed{(A_1 - 2I) v2 = v1} \ \mathrm{I} \\ A_1 v3 = -6 v3 \end{cases}$$

I $\to$
$$\left(\begin{array}{ccc|c} -7 & 1 & -1 & -1 \\ 15 & -2 & 3 & -6 \\ 7 & -1 & 1 & 1 \end{array}\right) \begin{matrix} \\ F2 \to 15F1 + 7F2 \\ F3 \to F1 + F3 \end{matrix} \left(\begin{array}{ccc|c} -7 & 1 & -1 & -1 \\ 0 & 1 & 6 & -57 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

$$\to \begin{cases} -7x + y - z = -1 \to -7x - 57 - 7z = -1 \to -7x = 56 + 7z \to x = -8 - z \\ y + 6z = -57 \to y = -57 - 6z \end{cases}$$

$$\to \bar{x} = (-8 - z; -57 - 6z; z) = z.(-1; -6; 1) + (-8; -57; 0)$$

Entonces tomo $z=0$ y $v_2 = (-8 \,;\, -57 \,;\, 0)$

Por lo tanto $\boxed{P = \begin{bmatrix} -1 & -8 & -1 \\ -6 & -57 & 2 \\ 1 & 0 & 1 \end{bmatrix}}$ $\boxed{J = \begin{bmatrix} 2 & 1 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & -6 \end{bmatrix}}$